Short Course
Interpretation of Pore Scale Experiments -
Alternative Descriptions of Porous Media
Flow by Topological Means

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With key contributions from
James McClure (Virginia Tech), Steffen Schlüter (UFZ Potsdam), Anna Herring (ANU), Christoph Arns (UNSW)
1. Motivation / necessity for and advantages of a geometric state function
   - 2-phase Darcy = phenomenological extension from 1-phase to 2-phase flow
   - example of having incomplete state variables: ideal gas equation of state

2. Introduction:
   - introducing Minkowski functionals
   - Hadwiger’s theorem, Gauss-Bonnet Theorem, Steiner’s formula

3. Description how we found it
   - general background: ganglion dynamics \(\rightarrow\) topological changes
   - beamline data of 2013 experiment: \(\rightarrow\) Hysteresis in Euler characteristic \(\chi\)
   - proof: (1) Hadwiger’s theorem, (2) 220000 LBM simulations

4. Software: Avizo, FIJI (BoneJ plugin), Matlab, Python, Dragonfly/DeepRocks

5. Applications,
   - hysteresis model for relative permeability
   - new route to pc and relative permeability
   - Digital Rock: validation of pore scale simulation techniques
   - wettability: description of contact angle as deficit curvature
Integrated Subsurface Workflow

- Prediction between wells
- Structure
- Reservoir Area
- Gross thickness
- Fluid distribution/contacts
- Net sand
- Porosity
- Saturation
- Reservoir pressure
- Compressibility
- Fluid PVT
- Aquifer
- Reservoir Connectivity
- Wells: Skin
- Wells Lift
- Production Data
- Statistics/ Uncertainty handling
- Capillary pressure
- Relative permeability
- Permeability
Multiphase Flow in Porous Media at Darcy Scale

**Single-Phase**

- Water

\[
\nu_{Darcy} = -\frac{K}{\mu} \frac{dp}{dx}
\]

**Two-Phase**

- Water, oil

\[
v_i = -k_{r,i} \frac{K}{\mu_i} \frac{dp_i}{dx}
\]

Phenomenological extension of Darcy’s law

### Darcy’s law

- \(\mu\) viscosity
- \(P\) pressure
- \(K (K_{abs})\) absolute permeability

Viscous law (similar to pipe flow) can be derived from upscaling
Stokes flow at pore scale by homogenization

### Relative Permeability

\[
k_{r,i} = k_{r,i}(S_w)
\]

**Capillary pressure**

\[
p_c = p_o - p_w
\]

![Diagram](image)
Historical Overview – Classification of the Problem

- **1930s: flow problem**: mass & momentum balance (Wykoff & Botset, Muskat & Meres, Leverett)
- **1970s**: thermodynamics of pore scale displacements (Morrow, Swanson & Yuan)
- **1990s**: equilibrium thermodynamics problem: mass, momentum & energy balance (Hassanizadeh & Gray) ganglion dynamics in 2D micromodels (Avraam & Payatakes)
- **2000s**: non-equilibrium thermodynamic problem – no global energy minimum TCAT (Gray & Miller)
- **2010s**: non-local dynamics, ganglion dynamics in 3D 
  \[ S_w + A_{nw} \] does not close pc hysteresis Armstrong, McClure et al. 2018
- **2018**: state variables: \( S_w, A_{nw}, p_c, \chi \)
- **2019**: wettability (upscaled contact angle as deficit Gaussian curvature)
Unresolved Issue: Capillary Pressure - Hysteresis

Capillary pressure function of saturation only \[ P_{nw} - P_w = P_c = f(S_w) \]

But: hysteresis

Hysteresis challenges the validity of 2-phase Darcy. But is it really hysteresis? What constitutive relationships properly represent multiphase flow?
Imagine we did not know about Temperature ... and only do P – V experiments, without measuring or controlling T ...

But with correct number of state variables, p, V, T, there is no hysteresis (for an ideal gas).

\[ p \cdot V = nRT \]

**Example: ideal gas, equation of state**

**Carnot cycle**
- Isothermal expansion / compression
- Adiabatic expansion / compression
Saturation $S_w$ and Interfacial Area $A_{nw}$ are State Variables of $P_c$

But that did not close the capillary hysteresis (McClure, Gray, Miller ...)

This term was not included in the traditional theories

\[ \Delta F = - S \Delta T - \sum_{\alpha=1}^{2} p_{\alpha} \Delta V_{\alpha} + \sigma_{1,2} \Delta A_{1,2} \]

Thermodynamics: free energy

\[ m_0 = \text{volume (saturation)} \]
\[ m_1 = \text{interfacial area} \]
\[ m_2 = \text{mean curvature} \]
\[ m_3 = \text{Gauss curvature} = 2\pi \chi \]

4 Minkowski Functionals

Morrow, 1970
The Source of Capillary Pressure Hysteresis

<table>
<thead>
<tr>
<th>Darcy Scale</th>
<th>Pore Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Macroscopic Darcy-scale “saturation functions”</td>
<td>f(pore scale fluid distribution)</td>
</tr>
</tbody>
</table>

\[ p_w = p_0 - p_w \]

Primary drainage

Secondary drainage

Imbibition

Residual oil sat. \( S_{or} \)

Irreducible water sat. \( S_{wc} \)

Water saturation \( S_w \)

= geometrical shape in 3D

[Stegemeier 1977]
The Source of Capillary Pressure Hysteresis

Darcy Scale

<table>
<thead>
<tr>
<th>Macroscopic Darcy-scale</th>
<th>=</th>
<th>Pore Scale</th>
</tr>
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<tr>
<td>“saturation functions”</td>
<td>=</td>
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</tbody>
</table>

Hadwiger’s Theorem:
Uniquely parameterized by 4 Minkowski Functionals
The Minkowski Functionals

Named after **Hermann Minkowski**, Mathematician (1864-1909)

Hadwiger’s theorem: unique characterization of 3D objects by 4 Minkowski functionals

\[ m_0 = \text{volume (saturation)} \]
\[ m_1 = \text{interfacial area} \]
\[ m_2 = \text{mean curvature (cap. pressure)} \]
\[ m_3 = \text{integral curvature} = 2\pi \chi \]

\[ M_0^n = \lambda(\Omega_n) = \int_{\Omega_n} dr \]
\[ M_1^n = \lambda(\Gamma_n) = \int_{\Gamma_n} dr \]
\[ M_2^n = \int_{\Gamma_n} \left( \frac{1}{R_1} + \frac{1}{R_2} \right) dr \]
\[ M_3^n = \int_{\Gamma_n} \frac{1}{R_1 R_2} dr \]

**Apply to Multiphase Flow**

McClure et al. Phys. Rev. Fluids, 2018


The Euler Characteristic

Named after Leonhard Euler, German Mathematician (1707-1783)

\[ M_3(X) = \int_{\delta X} \left[ \frac{1}{(r_1 r_2)} \right] ds = 2\pi \chi(\delta X) = 4\pi \chi(X) \]

- Euler Characteristic measures the bulk connectivity of an object

\[ X = \text{Objects} - \text{Loops} + \text{Voids} \]

Or a collection of objects:

\[ X (\text{OIL}) = -40 \]

... a cube has 6 faces

[Herring et al. 2012]
Leonhard Euler and the Seven Bridges of Königsberg

The city of Königsberg in Prussia (now Kaliningrad, Russia) was set on both sides of the Pregel River, and included two large islands - Kneiphof and Lomse - which were connected to each other, or to the two mainland portions of the city, by seven bridges. The problem was to devise a walk through the city that would cross each of those bridges once and only once (Source: Wikipedia)  
https://en.wikipedia.org/wiki/Seven_Bridges_of_K%C3%B6nigsberg

2 islands
separated by a river
7 bridges

Walk to cross each bridge only once?

L. Euler: not possible + proof
The Euler Characteristic

\[ M_3(X) = \int_{\delta X} \left[ \frac{1}{r_1 r_2} \right] ds = 2\pi \chi(\delta X) = 4\pi \chi(X) \]

- Euler Characteristic measures the **bulk connectivity** of an object

\[ \chi = \text{Objects} - \text{Loops} + \text{Voids} \]

Or a collection of objects:

\[ \chi(X(\text{OIL})) = -40 \]

[Herring et al. 2012]
(1) Hadwiger’s Theorem

A remarkable theorem is the ‘completeness’ of the Minkowski functionals proven 1957 by H. Hadwiger [21]. This characterization theorem asserts that any additive, motion-invariant and conditionally continuous functional \( \mathcal{M} \) is a linear combination of the \( d + 1 \) Minkowski functionals \( M_\nu \),

\[
\mathcal{M}(A) = \sum_{\nu=0}^{d} c_\nu M_\nu(A),
\]

with real coefficients \( c_\nu \) independent of \( A \). Motion-invariance of the functional means that the functional \( \mathcal{M} \) does not dependent on the location and orientation of the grain \( A \). Since quite often the assumption of a homogeneous and isotropic system is made in physics, motion-invariance is not a very restrictive constraint on the functional. Nevertheless, in the case where external fields are applied...
**Gauss-Bonnet Theorem**

Explains the relationship between Gaussian curvature and topology.

\[ \int_S \frac{1}{r_1 r_2} ds + \int_B k_g dl = 2\pi \chi(S) \]

Gaussian curvature

\[ k_f = \left( \frac{1}{r_1} \cdot \frac{1}{r_2} \right) \]

Mean curvature

\[ p_c(S_w) = \gamma \cdot \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \]

The curvature is either on the surface (Gaussian Curvature) or at the edges (Geodesic Curvature).
(3) Steiner’s Formula

Provides a means to identify relationships between the Minkowski functionals

\[ \lambda(X \oplus \delta r) - \lambda(X) = \sum_{i=1}^{3} a_i M_i r^i \]

Explains how the volume (dependent) of an object changes depending on the objects morphology (independent)

Example: sphere ...

\[ V(\Omega_i \oplus \delta \zeta) - V(\Omega_i) = \frac{4}{3} \pi (r + \delta r)^2 - \frac{4}{3} \pi (r)^2 = A_i \delta r + H_i (\delta r)^2 + \frac{4}{3} \pi \chi_i (\delta r)^3 \]

Mean width \[ H_i = \int_{\Gamma_i} \frac{\kappa_1 + \kappa_2}{2} dS \]

\[ A_i = 4\pi r^2 \]
\[ H_i = 4\pi r \]
\[ \chi_i = 1 \]
Background: Cluster Dynamics in SCAL Experiments

- Fractional flow
- Fluctuations
- Ganglion dynamics?

**Equation:**

\[ v_i = -k_{r,i} \frac{K}{\mu_i} \frac{dp_i}{dx} \]

- Rock sample 5 cm
- Saturation
- Resistivity
- Pressure

- 0.48 \(<\ Sw <\ 0.70

- Connected pathway flow
- Oil
- Water
- Imaged by fast \( \mu \)CT

- Rücker et al. GRL, 2015
- GRL, 2019
- Armstrong et al. WRR 2018

- Trapped oil ganglia

**Legend:**

- [Avraam & Payatakes, 1995]
Cluster Dynamics Introduces Topological Changes

1. H₂O
2. Oil
3. Break up
4. Coalescence

Strongly water-wet

Introduces Topological Changes

Rücker et al. GRL 2015
Cluster Dynamics Introduces Topological Changes

\[ \chi = \text{Objects} - \text{Loops} + \text{Voids} \]

Snap-off
→ disconnection
→ \#objects increases
→ \( \chi \) increases

Coalescence
→ connection
→ \#objects decreases
→ \( \chi \) decreases

Here: \#snap-off > \#coalescence
→ net \( \chi \) increase
The Discovery of the 4th State Variable

Oil-Saturation [%]

χ ∼ (S₀)²

Drainage = maintaining connectivity (avoid forming loops)

Imbibition = Snap-off → formation of clusters and loops

Rücker et al. 2015

Exponent depends on rock type

Schlüter et al. WRR, 2016

Hysteresis loop

bounding curve = drainage

Imbibition front

capillary finger
Proof by Direct Numerical Simulation

FIG. 6. Traditional models assume that the macroscale capillary pressure is a function only of the saturation of the wetting fluid. Two-fluid displacement simulations within a sand pack show that: (a) At fixed saturation, $s^w$, the relative mean curvature $\dot{f}_{wn}^w$ can attain many possible values depending on the system history; (b) $\dot{f}_{wn}^w(s^w, \epsilon^w)$ is non-unique for $s^w = 0.55$.

McClure et al. Phys. Rev. Fluids, 2018
Saturation and Saturation + Interfacial Area are not sufficient to fully parameterize hysteresis (error > 10%)
→ Saturation + Interfacial Area + Euler Characteristic: error < 10% → full set of state variables
Upscaling from Pore to the Darcy Scale

**“Pore scale”**
- Single pores
- Continuum oil & water phases
- Single interfaces

**“Cluster scale”**
- Non-wetting phase clusters,
  Cooperative dynamics
  \( \rightarrow \text{Changing topology} \)

**“Darcy scale”**
- Continuum mechanics: porosity, permeability, saturation
  Phenomenological description

\[ v_i = -k_{r,i} \frac{K}{\mu_i} \frac{dp_i}{dx} \]

**State variables:**
- Minkowski functionals

\[ \mu \text{ mm cm m} \]

Capillary

Viscous

\[ \mu m \text{ mm cm m} \]
How to Compute - Software Packages

0. Image processing
   1. Avizo
   2. ImageJ and FIJI: BoneJ plugin
   3. Python – scikit-image (currently only 2D)
   4. Matlab
   5. Quantim (https://www.ufz.de/export/data/2/94413_quantim4_ref_manual.pdf)
   6. Boundary and connectivity issues
   7. Dragonfly/DeepRocks – compute $\chi$ on network
Image Processing

**µCT image**  \(\xrightarrow{\text{segmentation}}\)  **Segmented**  \(\xrightarrow{\text{select phase}}\)  **Oil Phase**  \(\xrightarrow{\text{object analysis}}\)  **Oil clusters**

16 bit grey level (potentially filter)

Segmented image
- Blue = rock
- Red = oil
- Green = gas

For each **phase** compute
- Volume
- Interfacial area
- Mean curvature
- Euler characteristic

For each **cluster** compute
Avizo: Mean Curvature

segmentation

Label analysis
  • Volume
  • Area
  • Euler char.

Generate surface (spline interpolation)
  • Mean curvature
  • Gaussian curvature
Avizo: Surface meshes

- Surface property calculation
  - **Curvature**
  - Distance
  - Roughness
  - Thickness

[Image of surface meshes with Avizo software, courtesy of ThermoFisher]
Avizo : Object Analysis

• Analysis
  • Shape factor
  • Equivalent diameter
  • **Volume, area**
  • **Euler**
  • Orientation
  • Roundness
  • Sphericity
  • Rugosity
  • **Crofton**

• Classification

Grain separation on a 40GB Sandstone 5µm

[courtesy of ThermoFisher]
Avizo: Euler Characteristic

Euler Characteristic $\chi = \text{Objects} - \text{Loops (} + \text{Inclusions)}$

- $\chi = 1$
- $\chi = 2$
- $\chi = 0$
- $\chi = -1$
- $\chi = -4$

### Table

<table>
<thead>
<tr>
<th>Object</th>
<th>Volume</th>
<th>Area</th>
<th>BarX</th>
<th>Bary</th>
<th>Barz</th>
<th>Euler</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere</td>
<td>4169</td>
<td>1262.52</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>1</td>
</tr>
<tr>
<td>2 spheres</td>
<td>4169</td>
<td>1262.52</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>1</td>
</tr>
<tr>
<td>Sum</td>
<td>4170</td>
<td>1263.52</td>
<td>150</td>
<td>50</td>
<td>50</td>
<td>2</td>
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<td>Hollow sphere</td>
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<td>16953.9</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>2</td>
</tr>
<tr>
<td>Ring</td>
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<td>27631.2</td>
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<td>50</td>
<td>0</td>
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<tr>
<td>Doughnut</td>
<td>203899</td>
<td>49396.3</td>
<td>99.9872</td>
<td>50</td>
<td>50</td>
<td>-1</td>
</tr>
</tbody>
</table>

Herring, AWR 2013
ImageJ and FIJI: BoneJ plugin (OpenSource)

ImageJ - [https://imagej.nih.gov/ij/](https://imagej.nih.gov/ij/)
FIJI - [https://fiji.sc/](https://fiji.sc/)
BoneJ - [http://bonej.org/](http://bonej.org/)
ImageJ and FIJI: BoneJ plugin (OpenSource)

Euler Characteristic $\chi = \text{Objects} - \text{Loops} (+\text{Inclusions})$

<table>
<thead>
<tr>
<th></th>
<th>Euler char. ($\chi$)</th>
<th>Corrected Euler ($\chi + ...$)</th>
<th>Connectivity</th>
<th>Conn. density (pixel$^{-1}$)</th>
<th>Surface area (pixel$^{2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere.tif</td>
<td>1.0</td>
<td>1.0</td>
<td>0.0</td>
<td>0.0</td>
<td>1263.9727775132994</td>
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<td>-1.0</td>
<td>-5.0E-7</td>
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<td>-1.0</td>
<td>-1.0E-6</td>
<td>18358.24357045593</td>
</tr>
<tr>
<td>Ring.tif</td>
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<td>0.0</td>
<td>1.0</td>
<td>1.0E-6</td>
<td>29282.273575259434</td>
</tr>
<tr>
<td>doughnut.tif</td>
<td>-1.0</td>
<td>-1.0</td>
<td>2.0</td>
<td>1.0E-6</td>
<td>52570.373993201676</td>
</tr>
</tbody>
</table>

Herring, AWR 2013
Python

Euler Characteristic $\chi = \text{Objects} - \text{Loops (+ Inclusions)}$

$\chi = 1$
$\chi = 0$
$\chi = -1$
$\chi = 2$
$\chi = -4$

Many functions unfortunately currently only in 2D!

Herring, AWR 2013

https://scikit-image.org/docs/dev/api/skimage.measure.html
for w_idx = [900]  % ROI size
    w_surface = [];
    w_labels = [];
    i_loc = ((mask_x - w_idx)/2)+1;
    j_loc = ((mask_y - w_idx)/2)+1;
    k_loc = ((mask_z - w_idx)/2)+1;
    roi = FinalImage(i_loc:i_loc+w_idx-1, j_loc:j_loc+w_idx-1,
                     k_loc:k_loc+w_idx-1);
    [Euler_roi, Euler_labels] = imEuler3d(roi);
% Implements Euler number codes
    w_Euler = vertcat(w_Euler, Euler_roi);
    w_labels = vertcat(w_labels, Euler_labels);
    file_name = [num2str(w_idx), 'Filename_900.mat'];
    save(file_name, 'w_Euler', 'w_labels')
end

Boundary and Connectivity Issues

<table>
<thead>
<tr>
<th>ROI size</th>
<th>Euler3D calculated by Avizo</th>
<th>Euler3D calculated with MATLAB codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>-5.07E+02</td>
<td>-5.62E+02</td>
</tr>
<tr>
<td>300</td>
<td>-2.70E+03</td>
<td>-2.69E+03</td>
</tr>
<tr>
<td>400</td>
<td>-6.76E+03</td>
<td>-6.80E+03</td>
</tr>
</tbody>
</table>

Do these two objects connect?

YES $\rightarrow$ Euler = 1
No $\rightarrow$ Euler = 2
Measurements on Segmented Voxels

**Color and Sort by measurement**
- Volume
- Surface area
- Aspect Ratio
- Phi
- Theta
- Position (x,y,z)
- Intensity (mean, min, max, stdev)

**Digital Rock Imaging Platform**
Powered by Deep Learning

- **Whole Core**
  Medical CT, Core photography
- **Plugs and Cuttings**
  Micro-CT, Nano-CT, FIB-SEM
- **Thin Section**
  LM, SEM, TEM, CL, Mineralogy

*Courtesy of TheObjects*
Measurements Directly on Surface Meshes and Graphs

Mesh operations
- Smoothing
- Decimation
- Thickness
- Mean Curvature
- Gaussian Curvature

Pore network
- Nodes scaled by pore body radius
- Nodes colored by connectivity index
- Edges colored by edge length
- $\chi = -212$ (Euler characteristic)

Connectivity Histogram

Courtesy of TheObjects
Applications

1. Digital Rock: Validation of pore scale simulators: relative permeability
2. new route to relative permeability
3. Phase connectivity inferred from resistivity measurements
4. Phase connectivity in critical gas saturation
5. Hysteresis models for Darcy-scale flow
6. Wettability: Description of contact angle as deficit curvature
7. Wettability: bi-continuous interfaces in intermediate/mixed-wet rock
8. Petrology mineral analysis for interpreting relative permeability
9. Permeability in fracture networks
Application: Validation of Pore Scale Simulation

LBM fractional flow simulation: match with exp. data

Free-Energy LBM correctly predicts connectivity!

McClure et al., PRE, 2016

Alpak et al., ADWR 2018
Application: Validation of Pore Scale Simulation

Model: quasi-static Pore Network Model (Ruspini et al. 2018)
Rock: Gildehauser sandstone (similar to Bentheimer)
Imbibition

Bultreys et al. under review
Application: new route to relative permeability

Non-wetting phase relative permeability has high sensitivity with Euler characteristic.

Non-wetting phase relative permeability is simple Power law function of Euler characteristic.

Sensitivity

Bi-Continuous Interfaces

- Mean curvature $\sim 0$
- Gaussian curvature $< 0$

$\rightarrow$ bi-continuous interfaces with high connectivity

Resistivity Index and Topology

Percolation Theory:

\[ \xi \sim (p - p_c)^\beta \]

Percolation parameter = Saturation

Percolation parameter = Topology

Models comparing percolation parameters for various wetting conditions.

<table>
<thead>
<tr>
<th>Wettability</th>
<th>( \lg(\text{RI}) \sim \lg(S_w) )</th>
<th>( \lg(\text{RI}) \sim \lg(\frac{\chi_w}{\chi_p}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100% water-wet</td>
<td>0.98</td>
<td>0.22</td>
</tr>
<tr>
<td>50% water-wet</td>
<td>0.97</td>
<td>0.97</td>
</tr>
<tr>
<td>50% oil-wet</td>
<td>0.88</td>
<td>0.96</td>
</tr>
<tr>
<td>100% oil-wet</td>
<td>0.82</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Euler characteristic inferred from RI

[Liu et al. 2018]
Phase Connectivity in Critical Gas Saturation

Euler Characteristic $\chi$ = Objects – Loops (+ Inclusions)

$\chi = 1$

$\chi = 2$

$\chi = 0$

$\chi = -1$

$\chi = -4$

See SCA021 on Wednesday, 11:30
Advantage of state-variable description: path-independence

1. Can express any parameter, e.g. $k_r$, as total differential of state variables

\[
dk_r = \frac{\partial k_r}{\partial S} dS + \frac{\partial k_r}{\partial \dot{S}} d\dot{S} + \frac{\partial k_r}{\partial I} dI + \frac{\partial k_r}{\partial N_CA} dN_CA + \frac{\partial k_r}{\partial \lambda} d\lambda
\]

- Wettability
- Phase distribution
- Capillary number
- Rock structure

SPE-182655 (Penn State group)

2. Can choose different paths to measure $k_r$
   - e.g. branch 1: constant saturation (steady-state, constant fractional flow Fw)
   - branch 2: constant $p_c$ (porous plate)
Relative permeability as an Equation of State (EOS)

\[
\frac{\partial k_r}{\partial S} \frac{dS}{dt} + \frac{\partial k_r}{\partial \gamma} \frac{d\gamma}{dt} + \frac{\partial k_r}{\partial I} \frac{dI}{dt} + \frac{\partial k_r}{\partial N_{CA}} \frac{dN_{CA}}{dt} + \frac{\partial k_r}{\partial \lambda} \frac{d\lambda}{dt}
\]

- Phase distribution
- Capillary number
- Wettability
- Rock structure

\[
\frac{\partial k_r}{\partial S} = \frac{\partial S}{\partial t} > 0 \quad \frac{\partial S}{\partial t} < 0
\]

\[
\frac{\partial \gamma}{\partial S} = \alpha_x \left( \frac{\gamma - 1}{S - 1} \right) = \frac{1}{C_x (\gamma S)^{\alpha_x}}
\]

Graphs showing the relationship between Euler characteristic and Oleic phase saturation, and relative permeability.
Application: Wettability

Pore scale

Andrew et al. 2014

Darcy-SCAL scale

via capillary pressure curve

Effective contact angle ≠ intrinsic contact angle
(Intrinsic contact angle ~ surface energy)

Contact Angle from Deficit Curvature

Gauss-Bonnet Theorem

\[ \int_S \left[ \frac{1}{(r_1 r_2)} \right] ds + \int_B k_g dl = 2\pi \chi(S) \]

Total geodesic curvature along the contact line

Deficit Curvature, \( k_d \)

Macroscopic contact angle \( \theta_{macro} = \frac{k_d}{4N_c} \)
Contact Angle = Deficit Curvature

Gauss Bonnet Theorem

\[ 2\pi \chi(M) = \int_M \kappa_T dS + \int_{\partial M} \kappa_g dC \]

For each region:

Fluid-Fluid

\[ 2\pi \chi(M_{ab}) = \int_{M_{ab}} \kappa_T dS + \int_{\partial M_{ab}} \kappa_g dC, \]

Fluid-solid

\[ 2\pi \chi(M_{bs}) = \int_{M_{bs}} \kappa_T dS + \int_{\partial M_{bs}} \kappa_g dC. \]

\[ 4\pi \chi(C) = 2\pi \chi(M_{ab}) + 2\pi \chi(M_{bs}) \]

\[ = \int_{M_{ab}} \kappa_T dS + \int_{M_{bs}} \kappa_T dS + \int_{\partial M} (\kappa_{g_{ab}} + \kappa_{g_{bs}}) dC \]
Contact Angle = Deficit Curvature

\[ k_d = 4\pi \chi(C) - \kappa_{ab} A_{ab} - \kappa_{bs} A_{bs} \]

with

\[ \kappa_{ab} = \frac{1}{A_{ab}} \int_{M_{ab}} \kappa_T dS, \]
\[ \kappa_{bs} = \frac{1}{A_{bs}} \int_{M_{bs}} \kappa_T dS, \]

\[ k_d = \int_{\partial M} (\kappa_{gab} + \kappa_{gbs}) dC = \int_{\partial M} \kappa_{abs} n_{abs} \cdot [n_S \sin \theta + n_{bs}(1 - \cos \theta)] dC \]

For \( N_c \) contacts with solid

\[ k_d = \sum_{j=1}^{N_c} \int_{\partial M_j} \kappa_{abs} n_{abs} \cdot [n_S \sin \theta + n_{bs}(1 - \cos \theta)] dC \]

Macroscopic contact angle

\[ \theta^{macro} = \frac{k_d}{4N_c} \]

Armstrong et al. under review
Contact Angle = Deficit Curvature

Macroscopic contact angle

$$\theta_{macro} = \frac{k_d}{4N_c}$$

For 1 sessile droplet:

$$\kappa_{Gab} = -\sqrt{1-(r/R)^2}/r$$

$$\kappa_{Gbs} = 1/r$$

$$k_d = \int_{\partial M} \frac{1}{r} \left(1 - \sqrt{1-(r/R)^2}\right) dC$$

$$= 2\pi \left(1 - \sqrt{1 - (R \sin \theta/R)^2}\right)$$

$$= 2\pi (1 - \cos \theta).$$

$$\theta_{macro} = \frac{\pi (1 - \cos \theta)}{2}$$

From Eqs. (S9) and (S10)
Contact Angle = Deficit Curvature

Macroscopic contact angle

$$\theta_{macro} = \frac{k_d}{4N_c}$$

For 1 sessile droplet:

$$k_d = \int_{\partial M} \frac{1}{r} \left(1 - \sqrt{1 - (r/R)^2}\right) dC$$

$$= 2\pi \left(1 - \sqrt{1 - (R\sin \theta / R)^2}\right)$$

$$= 2\pi (1 - \cos \theta).$$

$$\theta_{macro} = \frac{\pi (1 - \cos \theta)}{2}$$

Armstrong et al. under review
Validation: Advancing and Receding Contact angle

\[ \theta_{macro} = \frac{k_d}{4N_c} \]

Morrow, JCPT 1975

Armstrong et al. under review
Predicting Contact Angle
from oil cluster distribution & $p_c(S_w)$
→ SCA009 (Tuesday, 9:30)

$$\theta_{macro} = \frac{\kappa_d}{4N_c}$$

Armstrong et al. *under review*
Minerology, Wettability and Topology

QEMSCAN DATA, Pore-Scale Minerology

Euler characteristic quantifies impact of wettability on the connectivity of the oil phase.
Fractured Porous Media

Original works of Adler (Fractured Porous Media)

PRL, Scholz et al. 2012

Dimensionless Density, $\rho'$
A measure of the connectivity of the fracture network

Fracture density x volume

Excluded volume

Number of Loops, $B_1 = [(\rho'/2)-1] \rho'$

[LOOPS + OBJECTS] / OBJECTS
Conclusions

- Prediction between wells
- Structure
- Reservoir Area
- Gross thickness
- Fluid distribution/contacts
- Net sand
- Porosity
- Saturation
- Permeability
- Relative permeability
- Capillary pressure
- Wells Lift
- Production Data
- Statistics/ Uncertainty handling
- Fractures
- Pchysteresis
- Relative permeability
- Wettability
- Oil pressure
- Capillary pressure
- PVT
- Wells: Skin
- Fluid saturation
Key Literature

Review Papers and Textbooks

Research Papers
Backup
2-Phase Flow in Porous Media

Darcy's law

\[ v_{Darcy} = -\frac{K}{\mu} \frac{dp}{dx} \]

Phenomenological extension of Darcy's law i.e. not a law anymore

\[ v_i = -k_{r,i} \frac{K}{\mu_i} \frac{dp_i}{dx} \]

\[ k_{r,i} = k_{r,i}(S_w) \]

\[ p_c = p_o - p_w = p_c(S_w) \]

Relative permeability

\[ k_r \]

\[ S_{or} \]

\[ S_{cw} \]

Capillary pressure

\[ \rho = \rho_o - \rho_w \]

Irreducible water sat. \( S_{wi} \)

2nd drainage

1st imbibition

Primary drainage

Scanning curve

Residual oil sat. \( S_{or} \)

Water saturation \( S_w \)
So far this has been sufficient, but

When trying to augment SCAL by Digital Rock, it is important to
• correctly classify the problem

And it becomes inevitable to understand what relative permeability actually is, i.e. face the
• Upscaling from Pore to Darcy Scale challenge
More conceptually.
The State Variables of Capillarity

Hadwiger’s theorem: unique characterization of 3D objects by 4 Minkowski functionals

- $m_0 = \text{volume (saturation)}$
- $m_1 = \text{interfacial area}$
- $m_2 = \text{mean curvature (cap. pressure)}$
- $m_3 = \text{integral curvature} = 2\pi\chi$

\[ M_0^n = \lambda(\Omega_n) = \int_{\Omega_n} dr \]
\[ M_1^n = \lambda(\Gamma_n) = \int_{\Gamma_n} dr \]
\[ M_2^n = \int_{\Gamma_n} \left( \frac{1}{R_1} + \frac{1}{R_2} \right) dr \]
\[ M_3^n = \int_{\Gamma_n} \frac{1}{R_1 R_2} dr \]

McClure et al. Phys. Rev. Fluids, 2018
Capillary Pressure vs. Saturation

**Capillary Pressure**

\[ P^c = \gamma J^{wn}_w \]

- \( P^c - S_w \) is essentially a geometric definition (or statement)
- Saturation does not uniquely define the geometrical state
- Steiner's formula suggests that all four MF are required for a unique definition

**MF in terms of macro-scale parameters**

\[ M_0^n = \varepsilon^n V \]

\[ M_1^n = (\varepsilon^{wn} + \varepsilon^{ns})V \]

\[ M_2^n = (J^{wn}_w \varepsilon^{wn} + J^{ns}_s \varepsilon^{ns})V \]

\[ M_3^n = \chi^n \]
Cluster Dynamics Introduces Topological Changes

Onset of cluster breakup at relatively oil saturation
Oil clusters remain mobile over large saturation range
Oil clusters immobilized / trapped close to $S_{or}$

Co-existence of connected pathway flow and ganglion dynamics over most of the mobile saturation range

[Rücker et al., GRL, 2015]
Characterization of Flow Regimes: Phase Diagrams …

Independent Variables: regions

- fractional flow $F_w = \frac{Q_w}{Q_w + Q_o}$
- $Ca = \frac{\mu v}{\sigma}$

Dependent Variables: trajectories

- Saturation $S_o = f(F_w, Ca)$
- $Ca = \frac{L_{cluster}}{r_{pore} \cdot \phi \cdot \mu v/\sigma}$
- State variables
  - clusters mobile
  - Breaking up
  - trapped clusters
  - mobilized by coalescence

Ganglion Dynamics

- classical

(at the time we did not know yet that we should have used $A_{nw}$)
Clusters: Growing and Coalescence $\rightarrow \chi$ decreases

- Growing Cluster (1)
- Coalescence (2)

$V(\text{cluster}) / V(\text{pores})$

$\text{time: 4min}$
$\text{time: 5min}$
$\text{time: 6min}$

$\text{time: 8min}$
$\text{time: 9min}$

Rücker et al. SCA2015-007
Clusters: Break-up by Snap-off $\rightarrow \chi$ increases

Rücker et al. SCA2015-007
Characterization of Flow Regimes: Phase Diagrams ...

Rücker et al. SCA2015-007
Application: Validation of Pore Scale Simulation

Can we obtain imbibition relative permeability from a quasi-static approach?


- MICP experiment
- GeoDICT drainage simulation Hg-air
- GeoDICT (scaled) o-w imbibition (data used for k_r simulation)

**large discrepancy**
Influence of Wettability on Topology

Dynamic Connectivity → [PNAS, Reynolds et al. 2017]

Power Associated with Flow

\[ P_i = \frac{dW}{dt} = -q_i \cdot \nabla p \approx \frac{\mu_i q_i^2}{K_i} \]

Surface Energy of WP/NWP Interface

\[ (Surface \ energy) \ E = \frac{\sigma}{l} \]

Tested Two Different Wettabilities

Water Wet: \( l = 0.72 \)

Mixed-Wet = -0.11